



Monday 23 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 Marks)

- 1 A sports car of mass 1.2 tonnes is being tested on a horizontal, straight section of road. After t s, the car has travelled x m from the starting line and its velocity is v m s⁻¹. The engine produces a driving force of 4000 N and the total resistance to the motion of the car is given by $\frac{40}{49}v^2$ N. The car crosses the starting line with speed 10 m s⁻¹.
- (i) Write down an equation of motion for the car and solve it to show that $v^2 = 4900 - 4800e^{-\frac{1}{735}x}$. [7]
- (ii) Hence find the work done against the resistance to motion over the first 100 m beyond the starting line. [5]
- 2 On a building site, a pulley system is used for moving small amounts of material up to roof level. A light pulley, which can rotate freely, is attached with its axis horizontal to the top of some scaffolding. A light inextensible rope hangs over the pulley with a counterweight of mass m_1 kg attached to one end. Attached to the other end of the rope is a bag of negligible mass into which m_2 kg of roof tiles are placed, where $m_2 < m_1$. This situation is shown in Fig. 2.

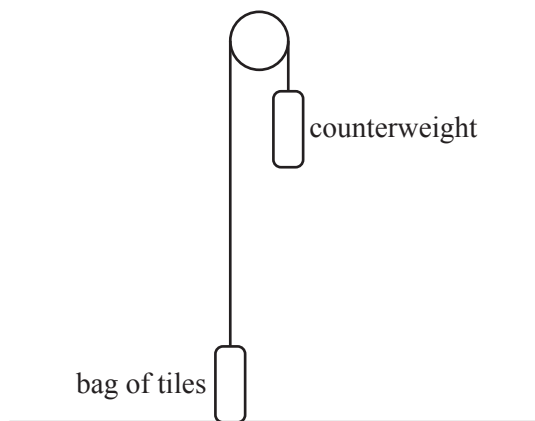


Fig. 2

Initially the system is held at rest with the rope taut, the counterweight at the top of the scaffolding and the bag of tiles on the ground. When the counterweight is released, the bag ascends towards the top of the scaffolding. At time t s the velocity of the counterweight is v m s⁻¹ downwards.

The counterweight is made from a bag of negligible mass filled with sand. At the moment the counterweight is released, this bag is accidentally ripped and after this time the sand drops out at a constant rate of λ kg s⁻¹.

- (i) Find the equation of motion for the counterweight while it still contains sand, and hence show that

$$v = gt + \frac{2gm_2}{\lambda} \ln \left(1 - \frac{\lambda t}{m_1 + m_2} \right). \quad [8]$$

- (ii) Given that the sand would run out after 10 seconds and that $m_2 = \frac{4}{5}m_1$, find the maximum velocity attained by the counterweight towards the ground. You may assume that the scaffolding is sufficiently high that the counterweight does not hit the ground before this velocity is reached. [4]

Section B (48 Marks)

- 3 A uniform rigid rod AB of mass m and length $2a$ is freely hinged to a horizontal floor at A. The end B is attached to a light elastic string of modulus λ and natural length $5a$. The other end of the string is attached to a small, light, smooth ring C which can slide along a horizontal rail. The rail is a distance $7a$ above the floor and C is always vertically above B. The angle that AB makes with the floor is θ . The system is shown in Fig. 3.

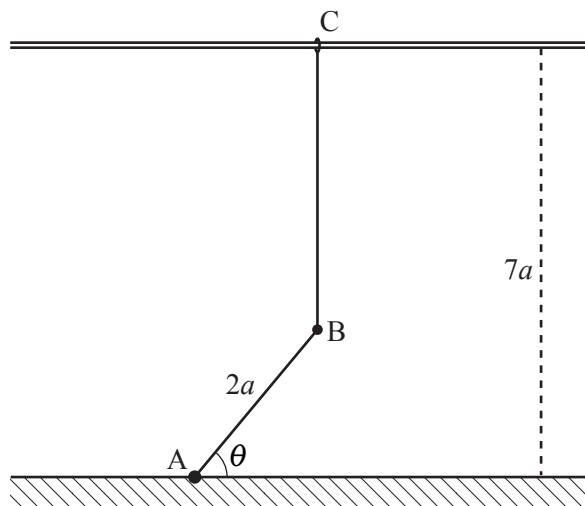


Fig. 3

- (i) Find the potential energy, V , of the system and hence show that

$$\frac{dV}{d\theta} = a \cos \theta \left(mg - \frac{4\lambda}{5} (1 - \sin \theta) \right). \quad [6]$$

- (ii) Show that there is a position of equilibrium when $\theta = \frac{1}{2}\pi$ and determine whether or not it is stable. [6]

There are two further positions of equilibrium when $0 < \theta < \pi$.

- (iii) Find the magnitude of the tension in the string and the vertical force of the hinge on the rod in these positions. [3]

- (iv) Show that $\lambda > \frac{5mg}{4}$. [4]

- (v) Show that these positions of equilibrium are stable. [4]

4

- 4 (a) A pulley consists of a central cylinder of wood and an outer ring of steel. The density of the wood is 700 kg m^{-3} and the density of the steel is 7800 kg m^{-3} . The pulley has a radius of 20 cm and is 10 cm thick (see Fig. 4.1).

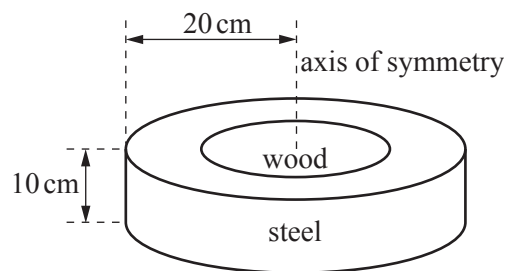


Fig. 4.1

Find the radius that the central cylinder must have in order that the moment of inertia of the pulley about the axis of symmetry shown in Fig. 4.1 is 1.5 kg m^2 . [6]

- (b) Two blocks P and Q of masses 10 kg and 20 kg are connected by a light inextensible string. The string passes over a heavy rough pulley of radius 25 cm. The pulley can rotate freely and the string does not slip. Block P is held at rest in smooth contact with a plane inclined at 30° to the horizontal, and block Q is at rest below the pulley (see Fig. 4.2).

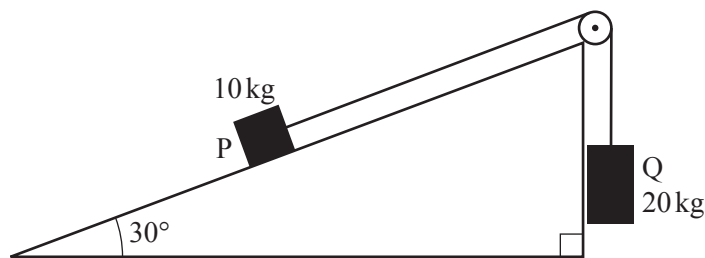


Fig. 4.2

At t s after the system is released from rest, the pulley has angular velocity ω rad s^{-1} and block P has constant acceleration of 2 m s^{-2} up the slope.

- (i) Show that the net loss of energy of the two blocks in the first t seconds of motion is $87t^2$ J and use the principle of conservation of energy to show that the moment of inertia of the pulley about its axis of rotation is $\frac{87}{32}$ kg m^2 . [6]

When $t=3$ a resistive couple is applied to the pulley. This resistive couple has magnitude $(2\omega + k)$ N m, where k is a constant. The couple on the pulley due to tensions in the sections of string is $\left(\frac{147}{4} - \frac{15}{8} \frac{d\omega}{dt}\right)$ N m in the direction of positive ω .

- (ii) Write down a first order differential equation for ω when $t \geq 3$ and show by integration that

$$\omega = \frac{1}{8} \left((45 + 4k) e^{\frac{64}{147}(3-t)} + 147 - 4k \right). \quad [8]$$

- (iii) By considering the equation given in part (ii), find the value or set of values of k for which the pulley

- (A) continues to rotate with constant angular velocity,
 (B) rotates with decreasing angular velocity without coming to rest,
 (C) rotates with decreasing angular velocity and comes to rest if there is sufficient distance between P and the pulley. [5]

END OF QUESTION PAPER